Table 1 Data of the combustion MHD generator

Fuel	JP-1
Oxydator	liquid oxygen
Seed material	potassium octoate (C_7H_{15}-COOK)
Seed rate	3% by weight of K in fuel + seed mat.
Total mass flow rate	1.5 kg/s
Combustion chamber	
Pressure	15 bar
Channel entrance	
Pressure	4.5 bar
Gas temperature	2850 K
Mach number	1.5
Cross-sectional area	20 cm^2
Distance from nozzle throat	11 cm
Channel exit	
Cross-sectional area	60 cm^2
Channel length	1.6 m

The experiments were performed on the 200 kW_{el} combustion MHD generator of the IPP-MAN association.[2] The data of the facility are presented in Table 1.
The channel, of the Hall type with circular cross section, was composed of axially segmented copper disks (9.5-mm thick). These segments were isolated both electrically and thermally from each other with 0.5-mm-thick insulating foils.

The density of the heat flux to the wall q_w was measured for various axial positions via the temperature increase in the channel wall as a function of time. All measurements were performed without an applied magnetic field. The measured values of q_w are plotted vs the distance x from the nozzle throat in Fig. 1.

Measurements were also performed with a diagonal wall channel with rectangular cross section. The entrance and exit cross-sectional areas and the length of this channel are the same as for the circular one. All the other data were very similar in the experiments with both channels. The values of q_w measured in the rectangular channel are also presented in Fig. 1.

A limit study was performed at Stanford for the conditions of the IPP experiment using the Patankar-Spalding[3] turbulent boundary-layer program. For comparison with the experimental values the results for the equilibrium and frozen limits in the case of a round channel are plotted in Fig. 1. (Rectangular channel results are virtually identical.) The behavior of the two theoretical distributions of q_w directly reflects the influence of the specific heat c_p. For the temperature at the channel entrance ($T \simeq 2850$ K) the finite-rate effect leads to a large difference between $c_{p,\text{frozen}}$ and $c_{p,\text{equ.}}$, whereas for the temperature in the exit region ($T \simeq 2500$ K) this difference is small. As can be seen, the data indicate that measured heat-transfer rates fall close to the frozen limit. It should be noted that catalytic wall effects would raise the heat transfer rate.

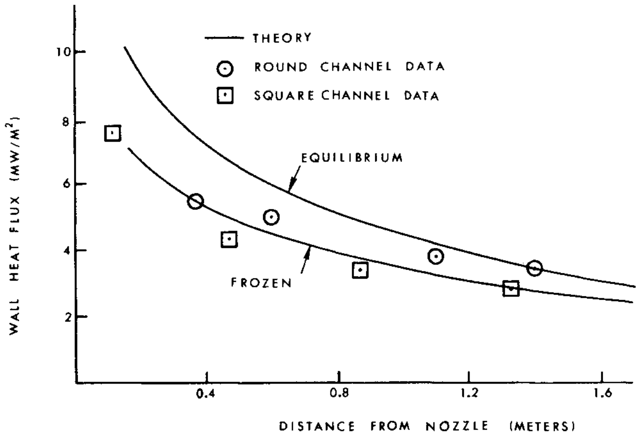

Fig. 1 Heat flux along the walls of the IPP-MAN round and square channels.

In conclusion, a substantial decrease in heat-transfer rate to the walls of a combustion-driven supersonic MHD power generator was observed which appears to occur because of chemical nonequilibrium in the developing wall boundary layers.

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Torque Development on a Spherical Body due to Spin Up of the Concentric Spherical Container

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Nomenclature

A = a function as defined in Eq. (24)
D = a parameter as defined in Eq. (28)
h = gap height between the two spheres, $R_o - R_i$
k = a parameter defined as $(\alpha/2\nu)^{1/2}$
N = viscous torque on the inner sphere
n = an integer variable
R_i = radius of the inner sphere
R_o = radius of the outer sphere
r = radial coordinate
t = time
v_r = velocity component in r-direction
v_θ = velocity component in θ-direction
v_ϕ = velocity component in ϕ-direction
x = $r - R_i$
z = Cartesian z-coordinate, selected along the axis of rotation of the spherical container
α = frequency in radians per unit time
β = $n\pi/h$
δ = a parameter defined in Eq. (29)
ε = phase angle in the harmonic motion of the spherical container
θ = angle θ in spherical coordinates
μ = viscosity of fluid
ν = kinematic viscosity, μ/ρ
ξ = a dummy variable for integration
ρ = density of fluid
τ = shear stress in fluid
ϕ = angle ϕ in spherical coordinates
ψ = a function as defined in Eq. (25)
Ω = angular speed of the outer sphere
Ω_o = maximum value of Ω
ω = angular speed of the fluid

Subscripts

r = r-direction in spherical coordinates
θ = θ-direction in spherical coordinates
ϕ = ϕ-direction in spherical coordinates

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I. Introduction

RECENTLY, fluid-supported spheres have drawn considerable attention not only because of their usefulness as stationary pivots with three degrees of freedom as in spherical bearings but also because of their applications in inertial guidance and control devices such as the use of a fluid-floated spinning sphere for momentum transfer in the attitude control of space vehicles and the use of a rotating free sphere as a stable reference in inertial guidance instrumentation. Spherical bearings are of special interest because they offer the advantages of relative insensitivity to misalignment and the possibility of combining the functions of both thrust and journal bearings. Besides, they are low-noise and low-cost bearings. Spherical cavities for such bearings readily lend themselves to manufacture by coining, while commercial balls may be useful for a bearing and are among the lowest cost high-precision mechanisms available. In such applications, often it is desired to study the development of torque on the fixed inner sphere due to a change in the steady-state condition of rest or angular rotation of the outer spherical container. The signal from the outer to the inner sphere is transmitted through the viscous fluid separating the two, and the torque on the inner sphere is caused by the viscous forces. It is therefore necessary to obtain a time-dependent solution of governing equations of the corresponding viscous flow within the bearing gap with appropriate initial and boundary conditions. In this paper, such a solution is obtained for concentric spheres and the viscous torque on the inner sphere is calculated. The results also provide the base from which the effect of eccentricity, inherent in any bearing problem, may be studied by perturbation techniques.

II. Theoretical Analysis

Consider a sphere of radius R_i acting as the stationary pivot within a concentric spherical container of radius R_o. Initially the system of these two spheres is at rest. At a certain reference time $t = 0$, the outer sphere suddenly starts spinning with an angular speed $\Omega(t)$ about the z-axis. Due to viscous effects, momentum is imparted to the fluid, which then starts rotating with an angular speed $\omega(t, r)$ about the axis of rotation of the spherical container, z-axis in the present case, and a torque is developed on the inner sphere due to viscous stresses created by the relative motion of the fluid. The problem here is to study the time-dependent motion of the fluid and to calculate the torque on the inner sphere. Only incompressible fluid of density ρ and viscosity μ is considered. As usual in bearing problems, the bearing gap height $h = R_o - R_i$ is assumed to be very small compared to the radii R_i and R_o. It is just pointed out that if initially the spherical container was spinning at a constant angular speed, then $\Omega(t)$ and $\omega(t, r)$ would represent the respective increments in the angular speeds of the sphere and the fluid.

Due to symmetry of the problem about the axis of rotation, z-axis in the present case,

$$\partial^m/\partial\phi^m = 0, \quad m = 1, 2, 3, \ldots$$

If it is assumed that the secondary flow is zero, then

$$v_r = v_\theta = 0$$

and the continuity equation is automatically satisfied. The Navier-Stokes equations for a viscous incompressible fluid in spherical coordinates then reduce to

$$\frac{\partial v_\phi}{\partial t} = \nu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_\phi}{\partial\theta}\right) - \frac{v_\phi}{r^2\sin^2\theta}\right] \quad (1)$$

The components of the shear stress tensor reduce to

$$\tau_{r\theta} = \tau_{rr} = \tau_{\theta\theta} = \tau_{\phi\phi} = 0$$

$$\tau_{\theta\phi} = \mu[\sin\theta(\partial v_\phi/\partial\theta) - v_\phi\cos\theta]/r\sin\theta$$

$$\tau_{\phi r} = \mu[r\sin\theta(\partial v_\phi/\partial r) - v_\phi\sin\theta]/r\sin\theta$$

The initial condition is

$$v_\phi(0, r, \theta) = 0 \quad (2)$$

Boundary conditions are

$$v_\phi(t, R_i, \theta) = 0 \quad (3)$$

$$v_\phi(t, R_o, \theta) = \Omega(t) R_o \sin\theta \quad (4)$$

Since the solution must be symmetric about the diametrical plane normal to the axis of rotation, the plane $z = 0$ in the present case, a trial solution of the form

$$v_\phi(t, r, \theta) = \omega(t, r) r\sin\theta \quad (5)$$

is assumed, where ω is the angular speed of the fluid. Then Eq. (1) reduces to

$$\partial\omega/\partial t = \nu[\partial^2\omega/\partial r^2 + (4/r)(\partial\omega/\partial r)] \quad (6)$$

with the initial condition

$$\omega(0, r) = 0 \quad (7)$$

and boundary conditions

$$\omega(t, R_i) = 0 \quad (8)$$

$$\omega(t, R_o) = \Omega(t) \quad (9)$$

The nonzero shear stress components now become

$$\tau_{\theta\phi} = 0, \qquad \tau_{\phi r} = \mu r\sin\theta \cdot \partial\omega/\partial r \quad (10)$$

The torque on the inner sphere due to viscous friction is calculated as

$$N = \int_o^\pi \underbrace{(\tau_{\phi r})_{r=R_i}}_{\text{Stress}} \underbrace{[(2\pi R_i\sin\theta)R_i\, d\theta]}_{\text{Area}} \underbrace{(R_i\sin\theta)}_{\text{Moment Arm}} = \tfrac{8}{3}\pi\mu R_i^4(\partial\omega/\partial r)_{r=R_i} \quad (11)$$

A new variable x is now introduced such that

$$x = r - R_i \quad (12)$$

Then the Eqs. (6–9) reduce to

$$\partial\omega/\partial t = \nu\{\partial^2\omega/\partial x^2 + [4/(R_i + x)](\partial\omega/\partial x)\} \quad (13)$$

$$\omega(0, x) = 0 \quad (14)$$

$$\omega(t, 0) = 0 \quad (15)$$

$$\omega(t, h) = \Omega(t) \quad (16)$$

where $h = R_o - R_i$ is the bearing gap height. Now, as $0 \leq x \leq h$ and because $R_i \gg h$, the second term on the right-hand side of Eq. (13) would be very small compared to the other terms and therefore can be dropped from the equation without changing its order. The equation then becomes

$$\partial\omega/\partial t = \nu(\partial^2\omega/\partial x^2) \quad (17)$$

The initial and boundary conditions remain the same as given in Eqs. (14–16). The expression for torque on the inner sphere reduces from Eq. (11) to

$$N(t) = \tfrac{8}{3}\pi\mu R_i^4(\partial\omega/\partial x)_{x=0} \quad (18)$$

The general solution[1] of Eq. (17) in the region $0 < x < h$ with the initial and boundary conditions given in Eqs. (14–16) is

$$\omega = -\frac{2\nu}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{\beta^2}{n}e^{-\nu\beta^2 t}\sin\beta x\int_o^t\Omega(\xi)e^{\nu\beta^2\xi}\,d\xi \quad (19)$$

where

$$\beta = n\pi/h \quad (20)$$

and ξ is the variable of integration. Once ω has been evaluated, the value of v_ϕ can be easily obtained from Eq. (5).

In general, the value of $\partial\omega/\partial x$ cannot be obtained simply by a term by term differentiation of Eq. (19) with respect to x because the resulting series will not always converge. In such cases, therefore, its value at any point in the fluid must be obtained by graphical or numerical methods using the expression for ω. Once the value of $(\partial\omega/\partial x)_{x=0}$ has been obtained this way, the viscous torque N on the inner sphere can be easily evaluated through Eq. (18). However, for certain functions $\Omega(t)$, Eq. (19) can be reduced to a sum of a simple function of x and t and a transient term, in which case $(\partial\omega/\partial x)_{x=0}$ may be evaluated directly from it by an ordinary differentiation with respect to x and then substituting $x = 0$ in the resulting expression. For example, consider the following two cases:

1. $\Omega = \Omega_o = \text{const}$

It is the case when the outer sphere suddenly starts spinning with a constant angular speed Ω_o about the z-axis due to some perturbation at time $t = 0$. Then from Eq. (19),

$$\omega = \frac{2\Omega_o}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \beta x (e^{-\nu\beta^2 t} - 1) \tag{21a}$$

which can be reduced to

$$\omega = \frac{\Omega_o x}{h} + \frac{2\Omega_o}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-\nu\beta^2 t} \sin \beta x \tag{21b}$$

The solution is seen to consist of a steady-state term and a transient which dies out as $t \to \infty$.

Differentiating Eq. (21b) with respect to x and then substituting $x = 0$ in the resulting expression, one obtains

$$\left(\frac{\partial \omega}{\partial x}\right)_{x=0} = \frac{\Omega_o}{h}\left[1 + 2 \sum_{n=1}^{\infty} (-1)^n e^{-\nu\beta^2 t}\right], \quad t > 0 \tag{22}$$

2. $\Omega = \Omega_o \sin(\alpha t + \varepsilon)$

It is the case when the outer sphere starts performing a harmonic motion about the z-axis with an initial angular speed $\Omega = \Omega_o \sin \varepsilon$ at $t = 0$. The solution, as obtained from Eq. (19), can be expressed as

$$\omega/\Omega_o = A \sin(\alpha t + \varepsilon + \psi) + 2\pi\nu \sum_{n=1}^{\infty} \frac{n(-1)^n(\nu n^2\pi^2 \sin \varepsilon - \alpha h^2 \cos \varepsilon)}{\nu^2 n^4 \pi^4 + \alpha^2 h^4} e^{-\nu\beta^2 t} \sin \beta x \tag{23}$$

where

$$A = \left[\frac{\cosh 2kx - \cos 2kx}{\cosh 2kh - \cos 2kh}\right]^{1/2} \tag{24}$$

$$\psi = \tan^{-1}\left(\frac{\tan kx}{\tanh kx}\right) - \tan^{-1}\left(\frac{\tan kh}{\tanh kh}\right) \tag{25}$$

and

$$k = (\alpha/2\nu)^{1/2} \tag{26}$$

The solution is again seen to consist of a steady-state term and a transient which dies out as $t \to \infty$.

Differentiating Eq. (23) with respect to x and then evaluating the resulting expression in the limit as $x \to 0$, the following is obtained:

$$\frac{1}{\Omega_o}\left(\frac{\partial \omega}{\partial x}\right)_{x=0} = D \sin(\alpha t + \varepsilon + \delta) + \frac{2\pi^2\nu}{h} \sum_{n=1}^{\infty} \frac{n^2(-1)^n(\nu n^2\pi^2 \sin \varepsilon - \alpha h^2 \cos \varepsilon)}{\nu^2 n^4 \pi^4 + \alpha^2 h^4} e^{-\nu\beta^2 t}, \quad t > 0 \tag{27}$$

where

$$D = 2k/(\cosh 2kh - \cos 2kh)^{1/2} \tag{28}$$

and

$$\delta = \pi/4 - \tan^{-1}(\tan kh/\tanh kh) \tag{29}$$

Now, the viscous torque N at any time $t > 0$ may be easily evaluated for the above two cases from Eq. (18) by substituting therein the proper expression for $(\partial\omega/\partial x)_{x=0}$.

III. Conclusions

A time-dependent solution of the viscous flow within the gap between two concentric spheres, when the inner sphere is fixed and the outer suddenly starts spinning about one of its axes, has been obtained under the assumption that the secondary flow is zero and that the gap width is small compared to the radii of the two spheres. Only an incompressible fluid with constant viscosity has been considered. An expression for the viscous torque on the inner sphere, which is also time-dependent, has been provided. The results may be considered the first approximation to the solution for the more general case involving eccentricity, the effect of which may be studied by perturbation techniques.

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Reversed Boundary-Layer Flows with Variable Fluid Properties

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In a comprehensive parametric study it is shown that the characteristics of reversed laminar boundary-layer flows are strongly influenced by the variations of the fluid transport properties. The existing data for constant density-viscosity flows are shown to be not only seriously in error relative to the results for flows with realistic variation of fluid properties but also to exhibit incorrect trends. The implications to corresponding turbulent flow calculations are noted.

Solutions of self-similar, constant density-viscosity product $[C = (\rho\mu)/(\rho\mu)_e = 1.0]$ fluid, laminar boundary-layer flows have been used extensively as the basic data for approximate analyses of flows with separation and reattachment (e.g., Lees and Reeves[1] and Gautier and Ginoux[2]). The use of computed boundary-layer profiles is considered to give improved accuracy over analyses employing simple polynomial representations of profiles. Since this approach is currently under consideration it seems appropriate to examine the relation of constant C data to the corresponding results based on more realistic fluid property variations.

Following the work of Stewartson,[3] Rogers[4] and Keller and Cebeci[5] have performed extensive studies of reversed boundary-layer flows of constant density-viscosity product fluids, with

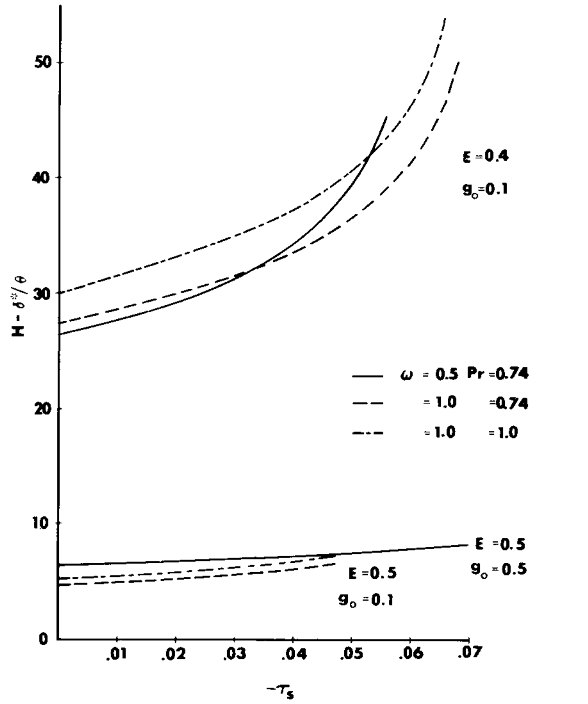

Fig. 1 Variation of the shape factor H with surface shear stress.

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Index category: Boundary Layers and Convective Heat Transfer—Laminar.

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